MINIMUM RETURN DIFFERENCE AS A COMPENSATOR DESIGN TOOL

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Supplementary materials

Note: I discovered a minor typo in the paper. On p. 126, run 3, the achieved steady-state error is 0.1, not 0.05. Note also that I should have explicitly mentioned in the paper, though it is obvious in Fig. 6a, upper figure, that without compensation the closed-loop system is unstable for the value of gain set for 0.1.
11/13/2001

1: EXPANDED FIG. 3 (on the following page).

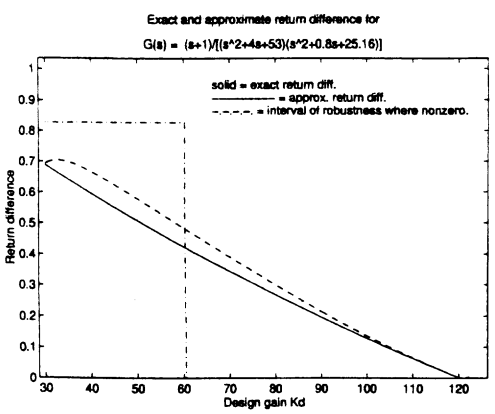
For brevity, the figures below were included for only one plant in the paper. Showing here results for a greater variety of plants demonstrates the validity of the RD_{min} approximation and enhances understanding of the relation between GM and PM (in particular, that that relation is plant-dependent).
Each plot in a row pertains to the same plant (in the plots, K_m is referenced by K_d):

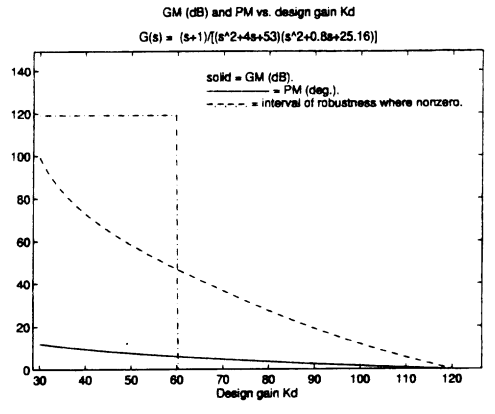
(a, left column) Approximate RD_{min} and exact RD_{min} vs. gain K_m.

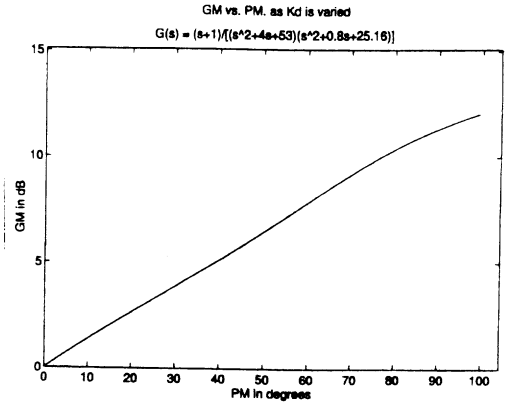
(b, center column) $GM(K_m)$ and $PM(K_m)$ vs. K_m.

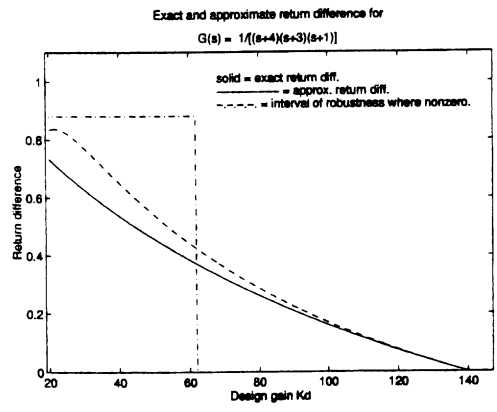
(c, right column) $GM(K_m)$ vs. $PM(K_m)$.

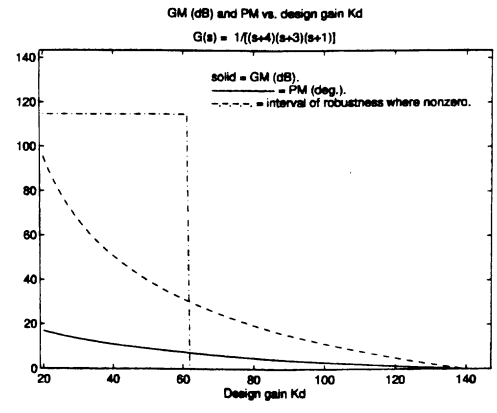
Note that numerous stylistic improvements have been made on the graphs in Fig. 3 of the paper relative to those shown here.

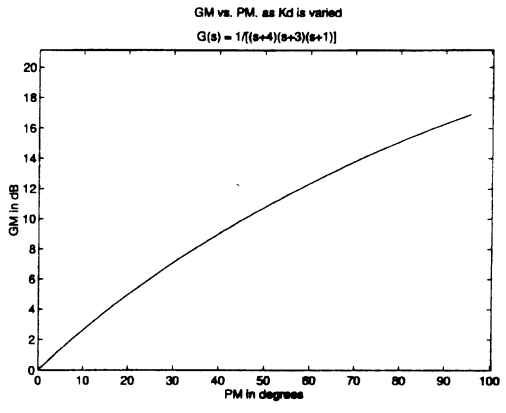

Exact and approximate return difference for
G(s) = (s+1)/[(s^2+4s+53)(s^2+0.8s+25.16)]
solid = exact return diff.
= approx. return diff.
= interval of robustness where nonzero.
Return difference
Design gain Kd

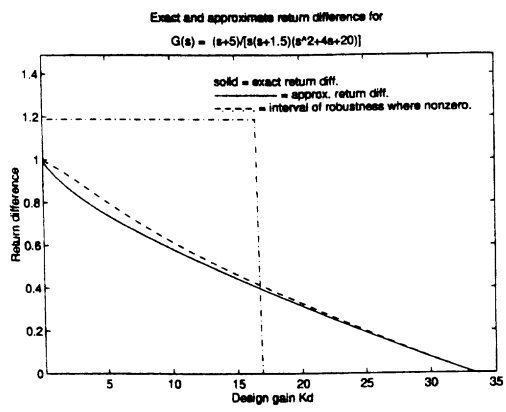

GM (dB) and PM vs. design gain Kd
G(s) = (s+1)/[(s^2+4s+53)(s^2+0.8s+25.16)]
solid = GM (dB).
= PM (deg.).
= interval of robustness where nonzero.
Design gain Kd

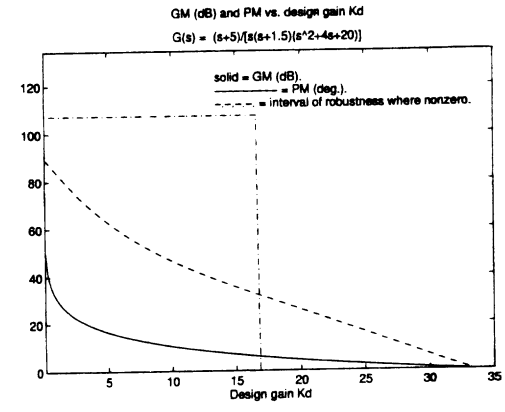

GM vs. PM. as Kd is varied
G(s) = (s+1)/[(s^2+4s+53)(s^2+0.8s+25.16)]
GM in dB
PM in degrees

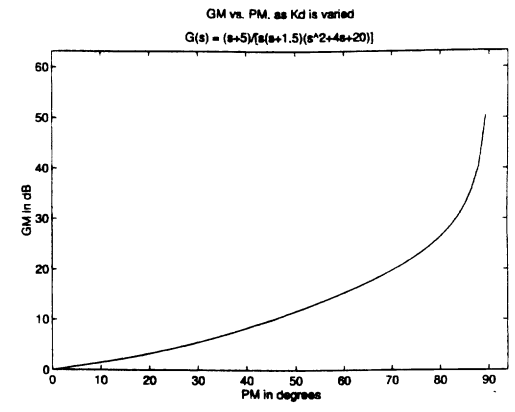

Exact and approximate return difference for
G(s) = 1/[(s+4)(s+3)(s+1)]
solid = exact return diff.
= approx. return diff.
= interval of robustness where nonzero.
Return difference
Design gain Kd

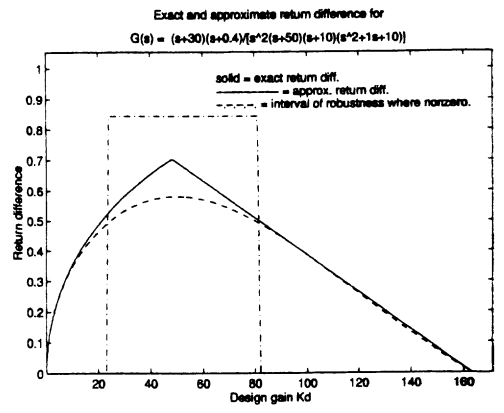

GM (dB) and PM vs. design gain Kd
G(s) = 1/[(s+4)(s+3)(s+1)]
solid = GM (dB).
= PM (deg.).
= interval of robustness where nonzero.
Design gain Kd

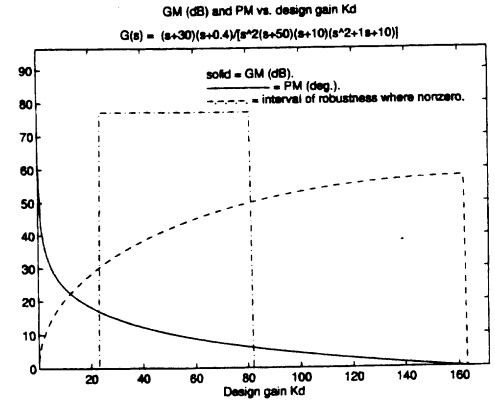

GM vs. PM. as Kd is varied
G(s) = 1/[(s+4)(s+3)(s+1)]
GM in dB
PM in degrees

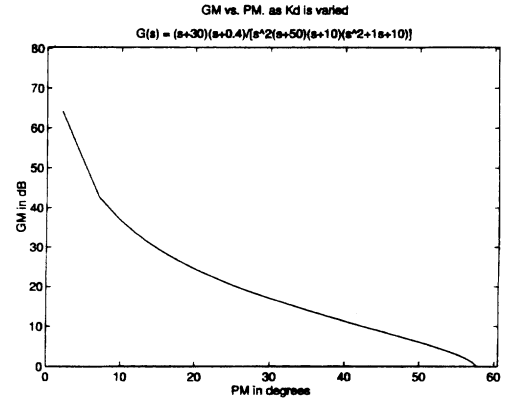

Exact and approximate return difference for
G(s) = (s+5)/[s(s+1.5)(s^2+4s+20)]
solid = exact return diff.
= approx. return diff.
= interval of robustness where nonzero.
Return difference
Design gain Kd


GM (dB) and PM vs. design gain Kd
G(s) = (s+5)/[s(s+1.5)(s^2+4s+20)]
solid = GM (dB).
= PM (deg.).
= interval of robustness where nonzero.
Design gain Kd

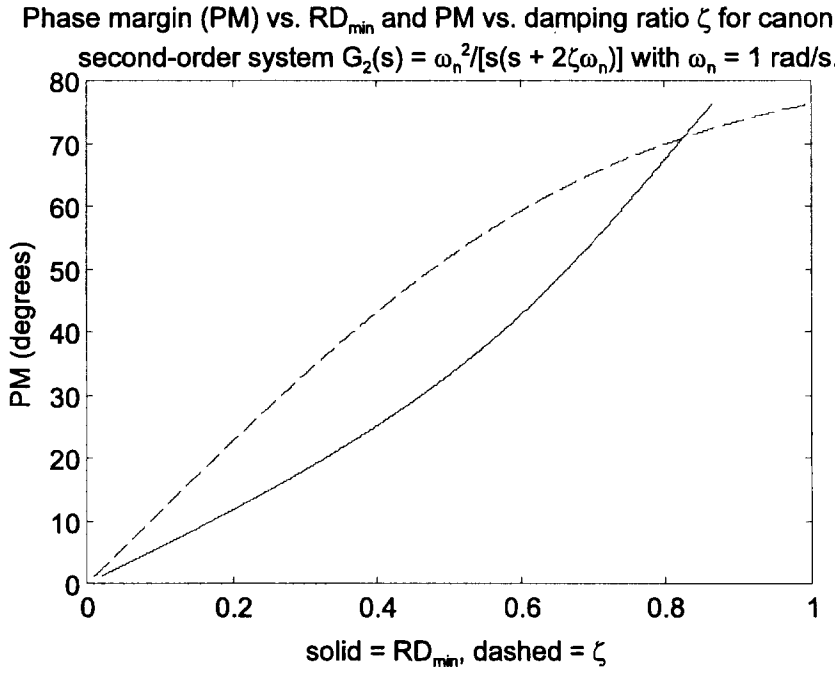

GM vs. PM. as Kd is varied
G(s) = (s+5)/[s(s+1.5)(s^2+4s+20)]
GM in dB
PM in degrees


Exact and approximate return difference for
G(s) = (s+30)(s+0.4)/[s^2(s+50)(s+10)(s^2+1s+10)]
solid = exact return diff.
= approx. return diff.
= interval of robustness where nonzero.
Return difference
Design gain Kd


GM (dB) and PM vs. design gain Kd
G(s) = (s+30)(s+0.4)/[s^2(s+50)(s+10)(s^2+1s+10)]
solid = GM (dB).
= PM (deg.).
= interval of robustness where nonzero.
Design gain Kd


GM vs. PM. as Kd is varied
G(s) = (s+30)(s+0.4)/[s^2(s+50)(s+10)(s^2+1s+10)]
GM in dB
PM in degrees

2: PLOT OF PM VS. RD_{min} AND OF PM vs. ζ FOR $G_2(s) = \omega_n^2/[s(s+2\zeta\omega_n)]$ with $\omega_n = 1$ rad/s and $K_m = 1$. Notice that PM rises with both RD_{min} and ζ, for $G(s) = G_2(s)$. Remember, however, that these plots are valid ONLY for the second-order canonical system with $K_m = 1$! Also, although ω_n was set to 1 rad/s to numerically specify $G_2(s)$ in the figure below, the value of ω_n does not affect the plots (ω_n affects only ω_0, ω_{gc}, and other critical frequencies via linear scaling of them).



3: EXPANDED FIG. 6 (on the following pages).

All these graphical simulation results are for the trials quantitatively described in Table I of the paper; Fig. 6 shows only two of these 20 runs, each of which may include both a lead and a lag design. For brevity, graphical results for all the other runs were omitted from the paper but are presented below. Showing here a greater variety of plants demonstrates the versatility of the RD-based compensator design algorithm and serves to compare it with the conventional PM-based Bode design procedures for lag and lead compensators.

Each plot in a row pertains to the same plant (in the plots, K_m is referenced by K_d):

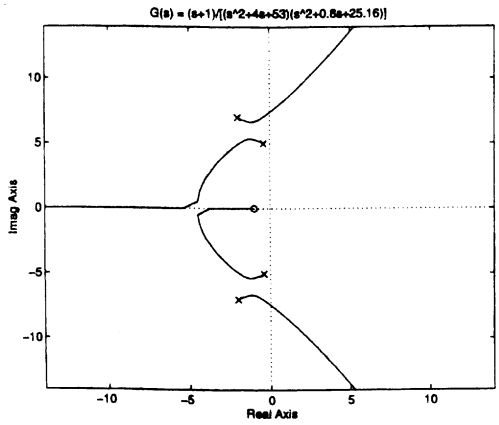
(a, left column) Root locus.

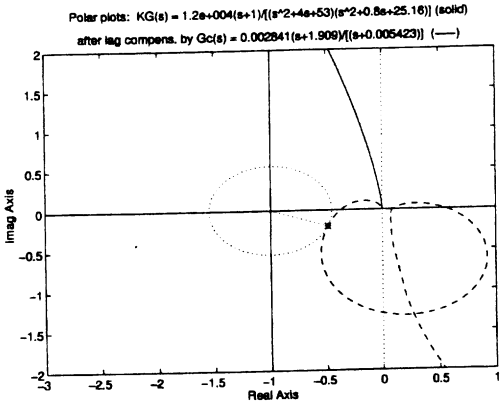
(b, center column) Polar plots of uncompensated and RD-compensated systems.

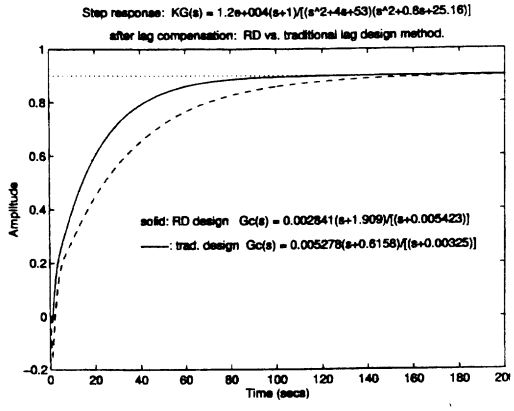
(c, right column) Closed-loop step response simulation comparisons.

Note that numerous stylistic improvements have been made on the graphs in Fig. 6 of the paper relative to those shown here.

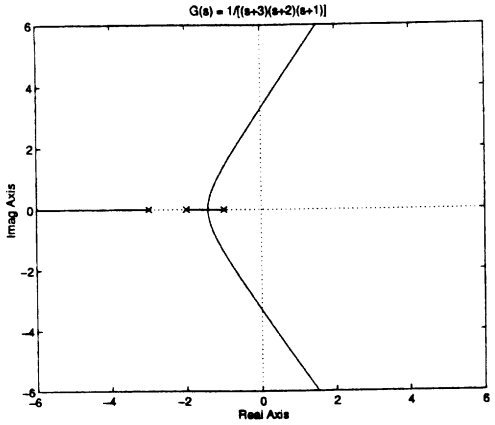
TF 1

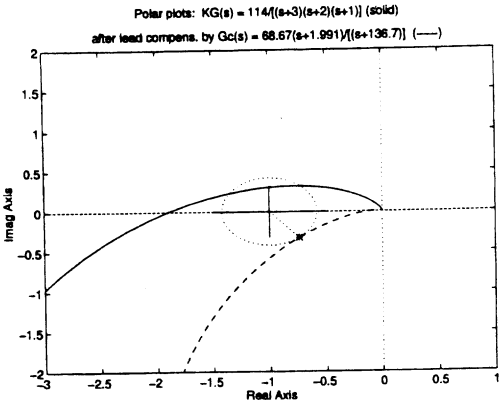

G(s) = (s+1)/[(s^2+4s+53)(s^2+0.8s+25.16)]
Imag Axis
Real Axis

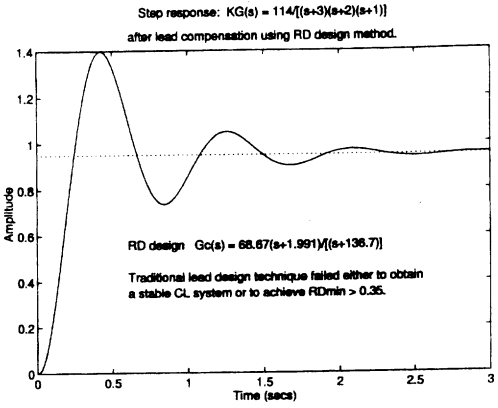

Polar plots: KG(s) = 1.2e+004(s+1)/[(s^2+4s+53)(s^2+0.8s+25.16)] (solid)
after lag compens. by Gc(s) = 0.002841(s+1.909)/[(s+0.005423)] (——)
Imag Axis
Real Axis

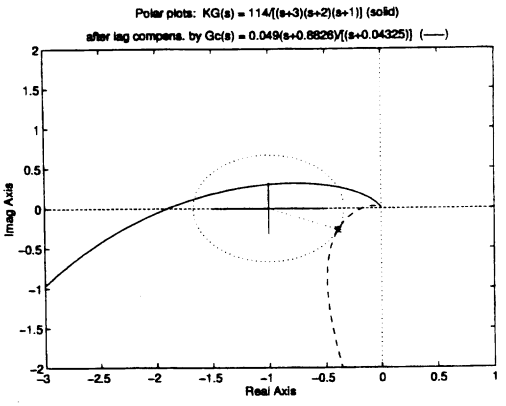

Step response: KG(s) = 1.2e+004(s+1)/[(s^2+4s+53)(s^2+0.8s+25.16)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.002841(s+1.909)/[(s+0.005423)]
——: trad. design Gc(s) = 0.005278(s+0.6158)/[(s+0.00325)]
Amplitude
Time (secs)

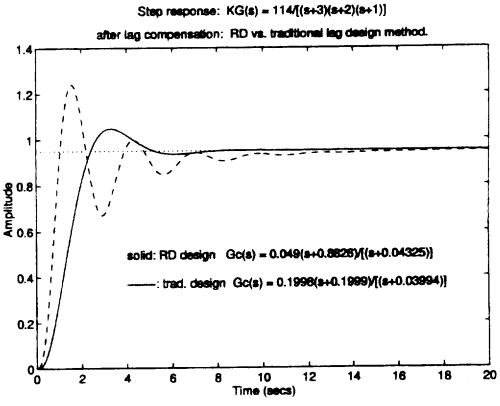
TF 2


G(s) = 1/[(s+3)(s+2)(s+1)]
Imag Axis
Real Axis

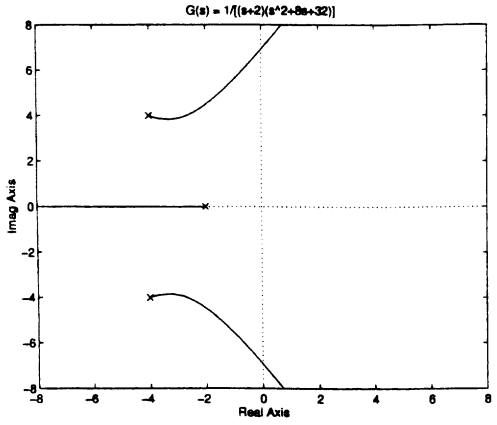

Polar plots: KG(s) = 114/[(s+3)(s+2)(s+1)] (solid)
after lead compens. by Gc(s) = 68.67(s+1.991)/[(s+136.7)] (——)
Imag Axis
Real Axis

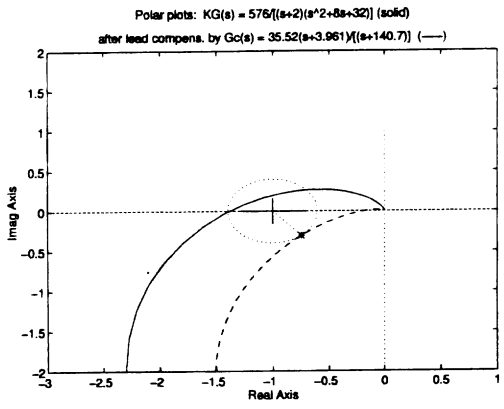

Step response: KG(s) = 114/[(s+3)(s+2)(s+1)]
after lead compensation using RD design method.
RD design Gc(s) = 68.67(s+1.991)/[(s+136.7)]
Traditional lead design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

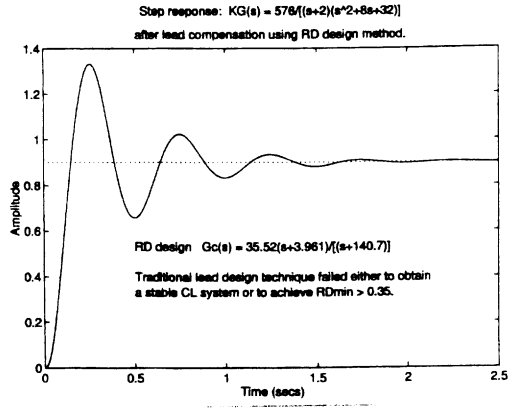

Polar plots: KG(s) = 114/[(s+3)(s+2)(s+1)] (solid)
after lag compens. by Gc(s) = 0.049(s+0.8826)/[(s+0.04325)] (——)
Imag Axis
Real Axis

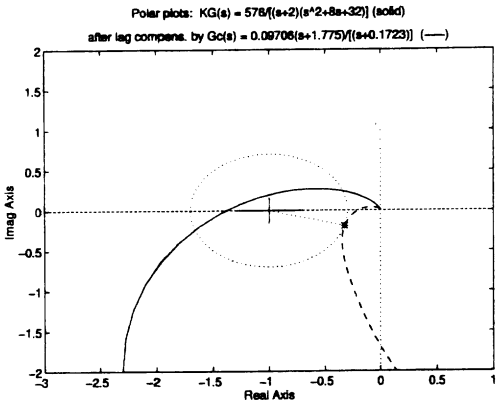

Step response: KG(s) = 114/[(s+3)(s+2)(s+1)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.049(s+0.8826)/[(s+0.04325)]
——: trad. design Gc(s) = 0.1998(s+0.1999)/[(s+0.03994)]
Amplitude
Time (secs)

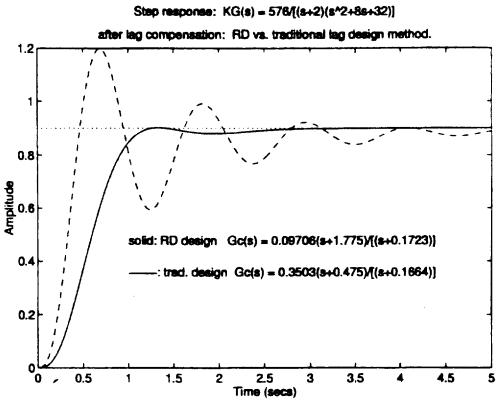
TF 3


G(s) = 1/[(s+2)(s^2+8s+32)]
Imag Axis
Real Axis

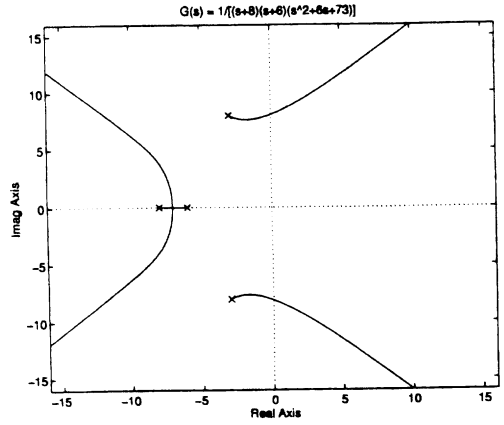

Polar plots: KG(s) = 576/[(s+2)(s^2+8s+32)] (solid)
after lead compens. by Gc(s) = 35.52(s+3.961)/[(s+140.7)] (——)
Imag Axis
Real Axis

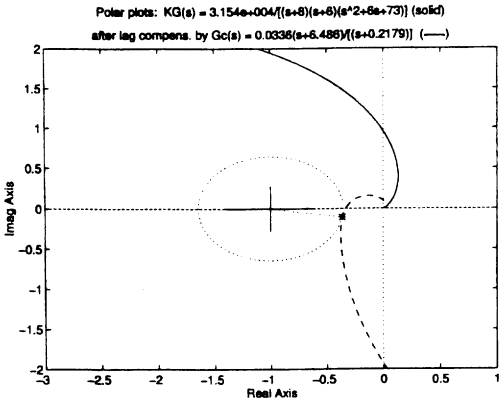

Step response: KG(s) = 576/[(s+2)(s^2+8s+32)]
after lead compensation using RD design method.
RD design Gc(s) = 35.52(s+3.961)/[(s+140.7)]
Traditional lead design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

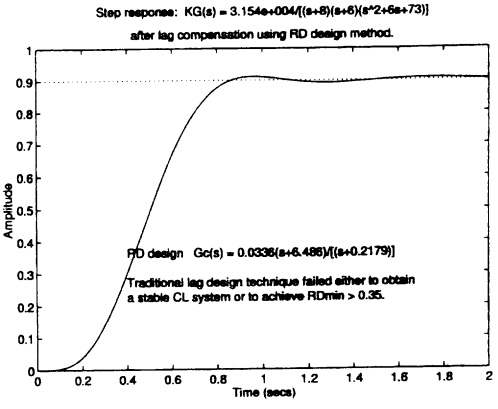

Polar plots: KG(s) = 576/[(s+2)(s^2+8s+32)] (solid)
after lag compens. by Gc(s) = 0.09706(s+1.775)/[(s+0.1723)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 576/[(s+2)(s^2+8s+32)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.09706(s+1.775)/[(s+0.1723)]
——: trad. design Gc(s) = 0.3503(s+0.475)/[(s+0.1864)]
Amplitude
Time (secs)

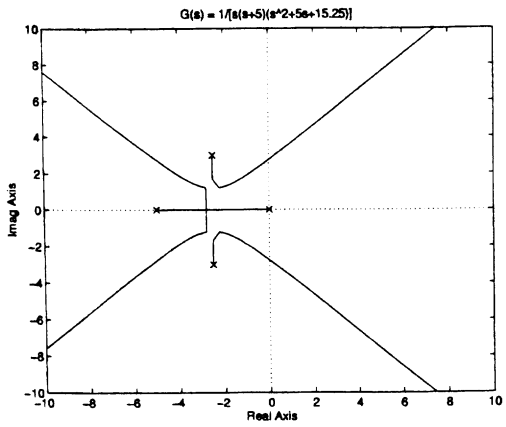
TF 4

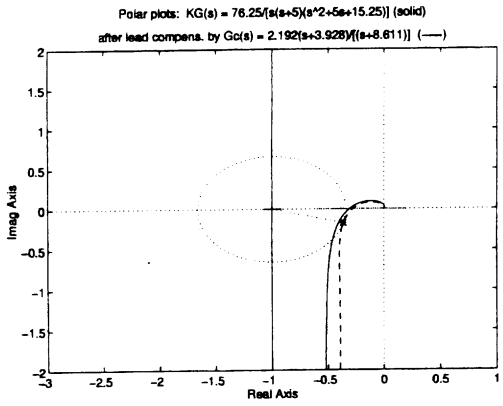

G(s) = 1/[(s+8)(s+6)(s^2+8s+73)]
Imag Axis
Real Axis

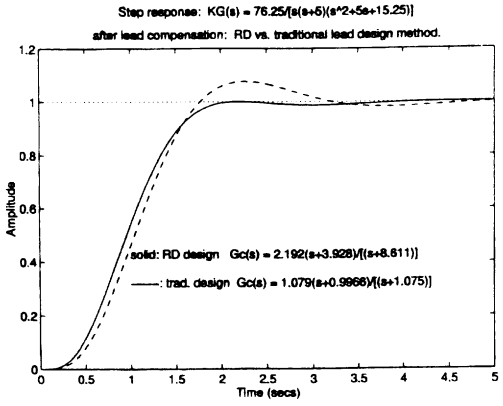

Polar plots: KG(s) = 3.154e+004/[(s+8)(s+6)(s^2+8s+73)] (solid)
after lag compens. by Gc(s) = 0.0336(s+6.486)/[(s+0.2179)] (——)
Imag Axis
Real Axis

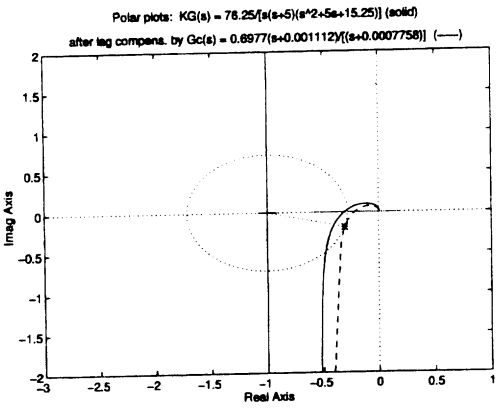

Step response: KG(s) = 3.154e+004/[(s+8)(s+6)(s^2+8s+73)]
after lag compensation using RD design method.
RD design Gc(s) = 0.0336(s+6.486)/[(s+0.2179)]
Traditional lag design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

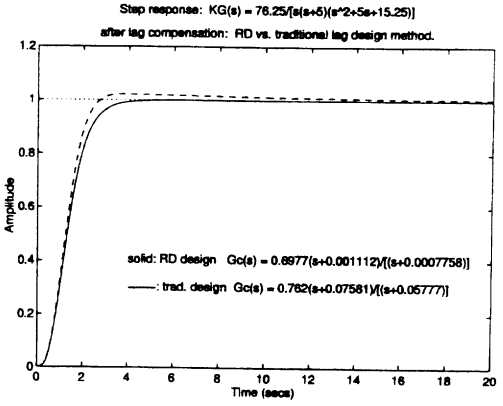
TF 5

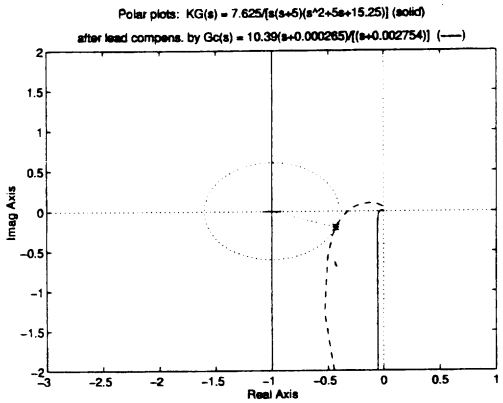

G(s) = 1/[s(s+5)(s^2+5s+15.25)]
Imag Axis
Real Axis

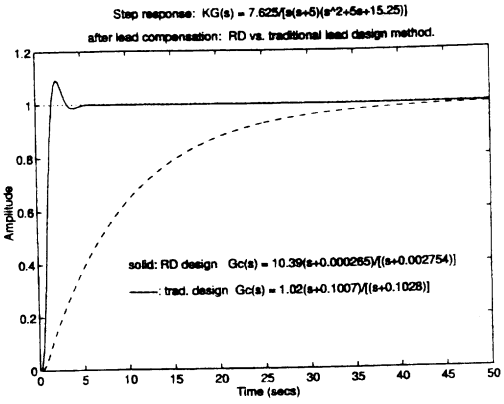

Polar plots: KG(s) = 76.25/[s(s+5)(s^2+5s+15.25)] (solid)
after lead compens. by Gc(s) = 2.192(s+3.928)/[(s+8.611)] (——)
Imag Axis
Real Axis

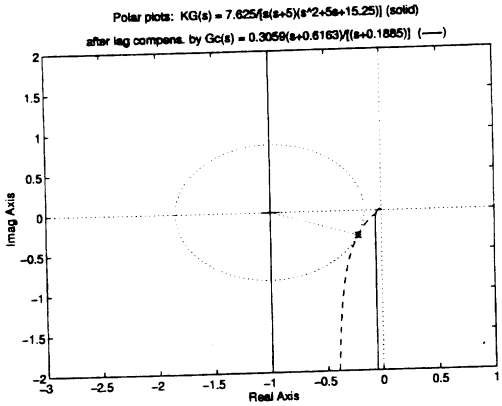

Step response: KG(s) = 76.25/[s(s+5)(s^2+5s+15.25)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 2.192(s+3.928)/[(s+8.611)]
——: trad. design Gc(s) = 1.079(s+0.9966)/[(s+1.075)]
Amplitude
Time (secs)

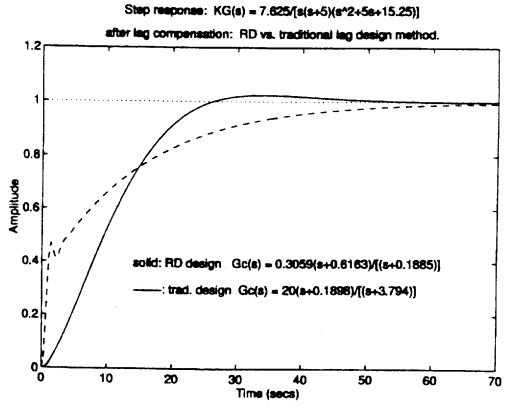

Polar plots: KG(s) = 76.25/[s(s+5)(s^2+5s+15.25)] (solid)
after lag compens. by Gc(s) = 0.6977(s+0.001112)/[(s+0.0007758)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 76.25/[s(s+5)(s^2+5s+15.25)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.6977(s+0.001112)/[(s+0.0007758)]
——: trad. design Gc(s) = 0.762(s+0.07581)/[(s+0.05777)]
Amplitude
Time (secs)

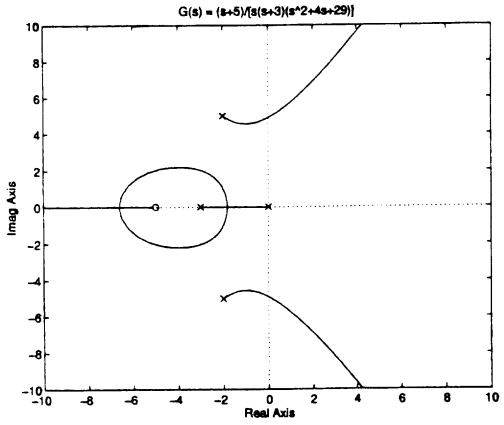

Polar plots: KG(s) = 7.625/[s(s+5)(s^2+5s+15.25)] (solid)
after lead compens. by Gc(s) = 10.39(s+0.000265)/[(s+0.002754)] (——)
Imag Axis
Real Axis

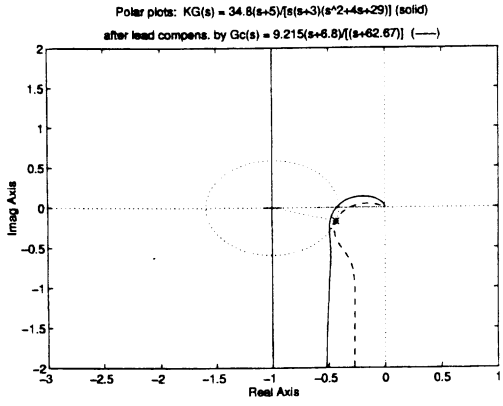

Step response: KG(s) = 7.625/[s(s+5)(s^2+5s+15.25)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 10.39(s+0.000265)/[(s+0.002754)]
——: trad. design Gc(s) = 1.02(s+0.1007)/[(s+0.1028)]
Amplitude
Time (secs)

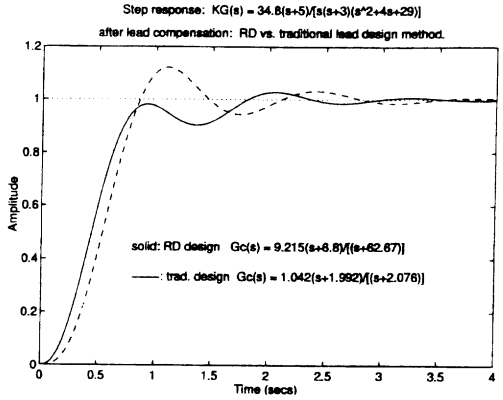

Polar plots: KG(s) = 7.625/[s(s+5)(s^2+5s+15.25)] (solid)
after lag compens. by Gc(s) = 0.3059(s+0.6163)/[(s+0.1885)] (——)
Imag Axis
Real Axis

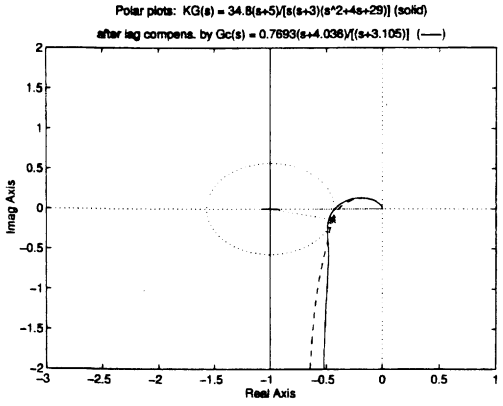

Step response: KG(s) = 7.625/[s(s+5)(s^2+5s+15.25)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.3059(s+0.6163)/[(s+0.1885)]
——: trad. design Gc(s) = 20(s+0.1898)/[(s+3.794)]
Amplitude
Time (secs)

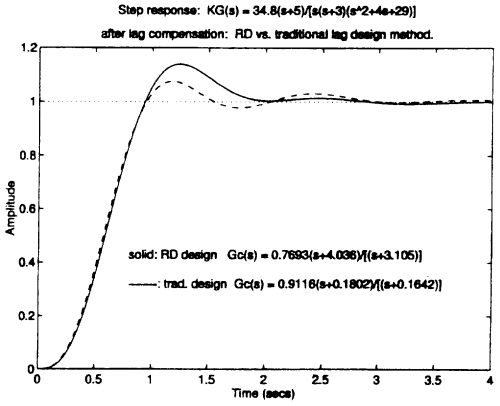
TF 6

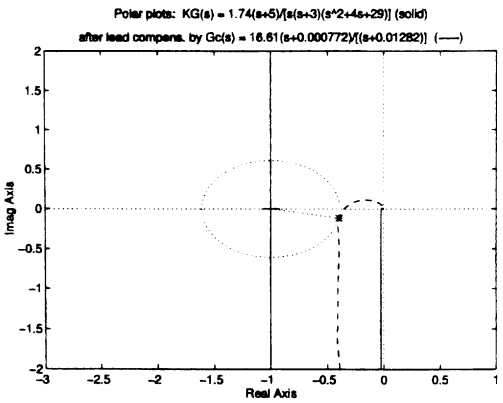

G(s) = (s+5)/[s(s+3)(s^2+4s+29)]
Imag Axis
Real Axis

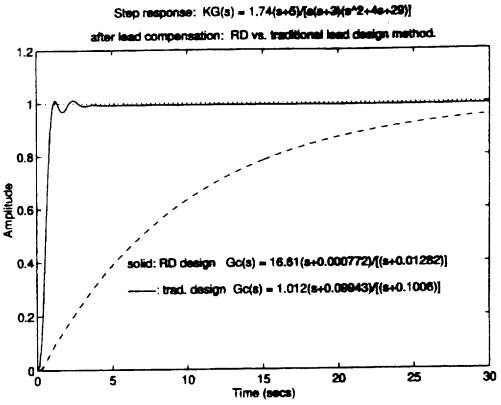

Polar plots: KG(s) = 34.8(s+5)/[s(s+3)(s^2+4s+29)] (solid)
after lead compens. by Gc(s) = 9.215(s+6.8)/[(s+62.67)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 34.8(s+5)/[s(s+3)(s^2+4s+29)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 9.215(s+6.8)/[(s+62.67)]
——: trad. design Gc(s) = 1.042(s+1.992)/[(s+2.076)]
Amplitude
Time (secs)

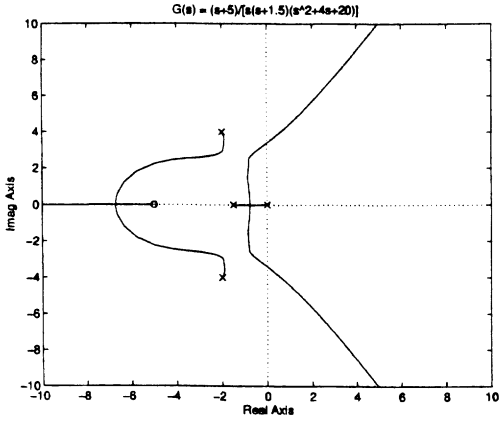

Polar plots: KG(s) = 34.8(s+5)/[s(s+3)(s^2+4s+29)] (solid)
after lag compens. by Gc(s) = 0.7693(s+4.036)/[(s+3.105)] (——)
Imag Axis
Real Axis

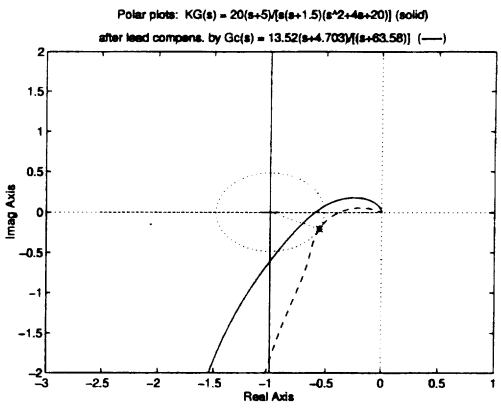

Step response: KG(s) = 34.8(s+5)/[s(s+3)(s^2+4s+29)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.7693(s+4.036)/[(s+3.105)]
——: trad. design Gc(s) = 0.9116(s+0.1802)/[(s+0.1642)]
Amplitude
Time (secs)

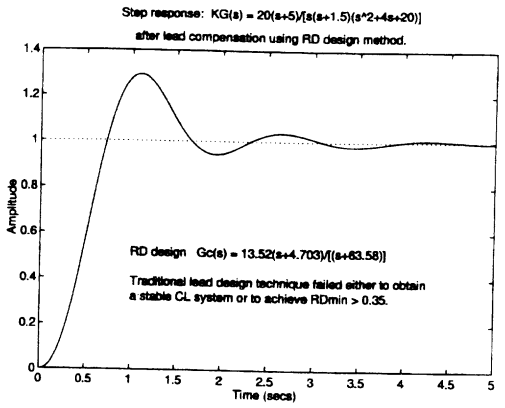

Polar plots: KG(s) = 1.74(s+5)/[s(s+3)(s^2+4s+29)] (solid)
after lead compens. by Gc(s) = 16.61(s+0.000772)/[(s+0.01282)] (——)
Imag Axis
Real Axis

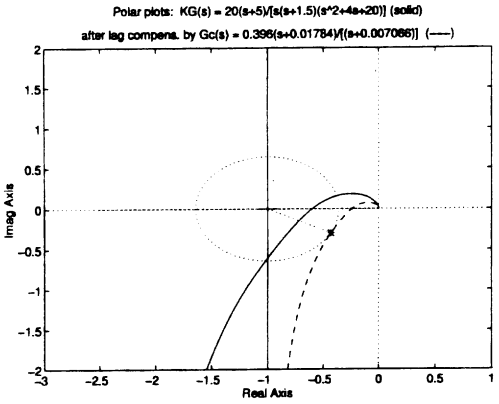

Step response: KG(s) = 1.74(s+5)/[s(s+3)(s^2+4s+29)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 16.61(s+0.000772)/[(s+0.01282)]
——: trad. design Gc(s) = 1.012(s+0.09943)/[(s+0.1006)]
Amplitude
Time (secs)

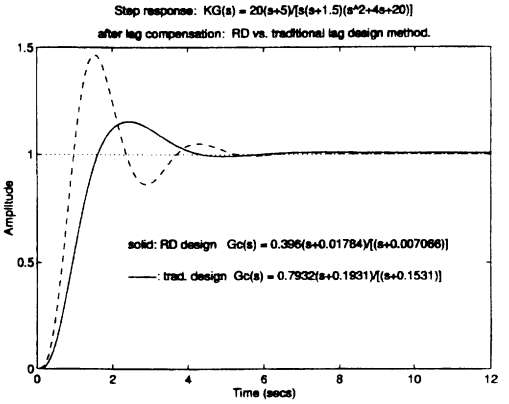
TF 7


G(s) = (s+5)/[s(s+1.5)(s^2+4s+20)]
Imag Axis
Real Axis

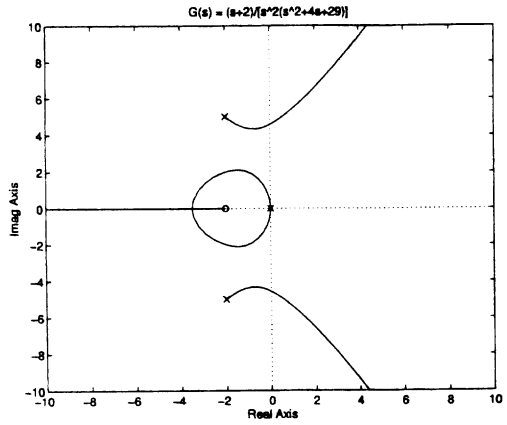

Polar plots: KG(s) = 20(s+5)/[s(s+1.5)(s^2+4s+20)] (solid)
after lead compens. by Gc(s) = 13.52(s+4.703)/[(s+63.58)] (——)
Imag Axis
Real Axis

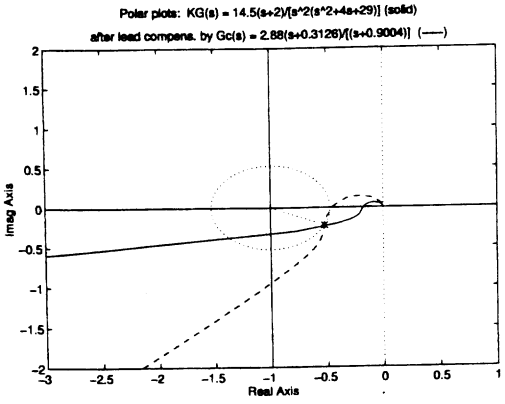

Step response: KG(s) = 20(s+5)/[s(s+1.5)(s^2+4s+20)]
after lead compensation using RD design method.
RD design Gc(s) = 13.52(s+4.703)/[(s+63.58)]
Traditional lead design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

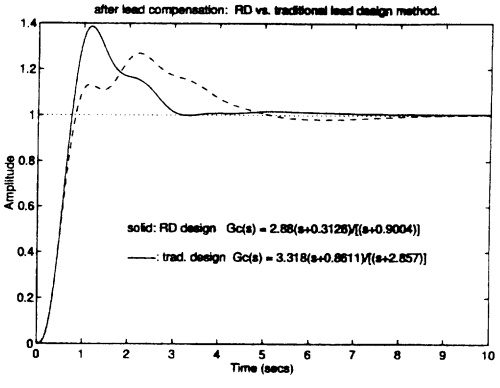

Polar plots: KG(s) = 20(s+5)/[s(s+1.5)(s^2+4s+20)] (solid)
after lag compens. by Gc(s) = 0.396(s+0.01784)/[(s+0.007066)] (——)
Imag Axis
Real Axis

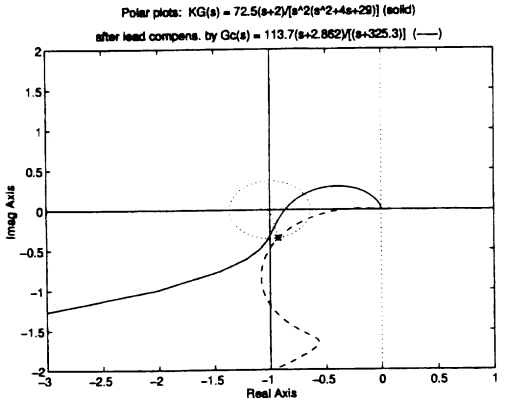

Step response: KG(s) = 20(s+5)/[s(s+1.5)(s^2+4s+20)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.396(s+0.01784)/[(s+0.007066)]
——: trad. design Gc(s) = 0.7932(s+0.1931)/[(s+0.1531)]
Amplitude
Time (secs)

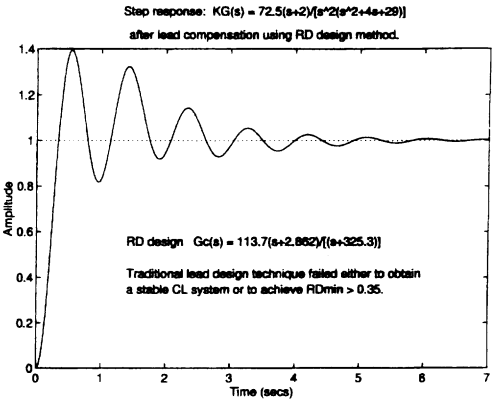
TF 8

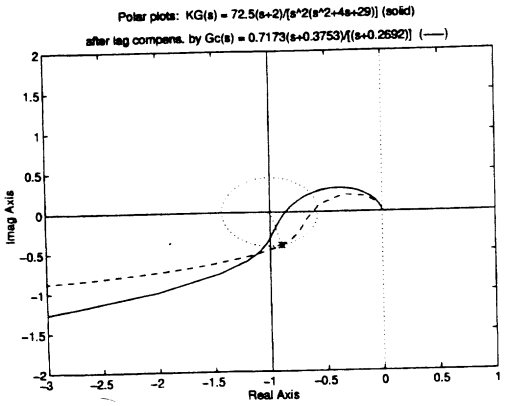

G(s) = (s+2)/[s^2(s^2+4s+29)]
Imag Axis
Real Axis

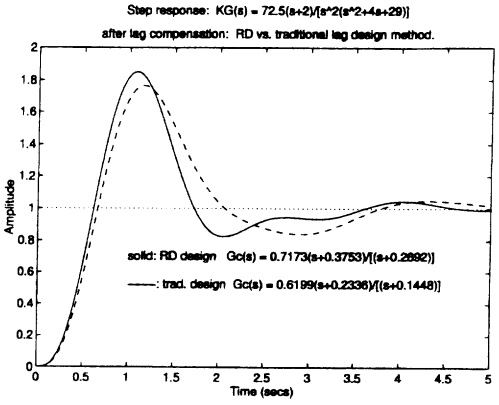

Polar plots: KG(s) = 14.5(s+2)/[s^2(s^2+4s+29)] (solid)
after lead compens. by Gc(s) = 2.88(s+0.3126)/[(s+0.9004)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 14.5(s+2)/[s^2(s^2+4s+29)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 2.88(s+0.3126)/[(s+0.9004)]
——: trad. design Gc(s) = 3.318(s+0.8611)/[(s+2.857)]
Amplitude
Time (secs)

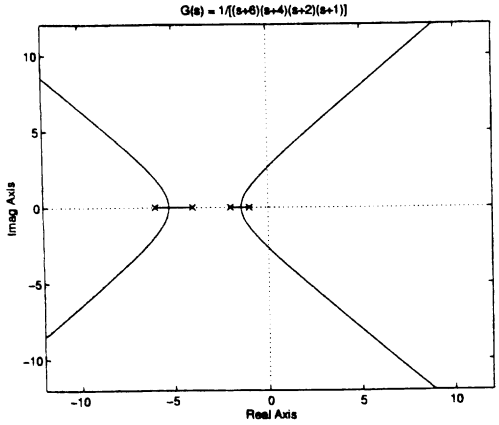

Polar plots: KG(s) = 72.5(s+2)/[s^2(s^2+4s+29)] (solid)
after lead compens. by Gc(s) = 113.7(s+2.862)/[(s+325.3)] (——)
Imag Axis
Real Axis

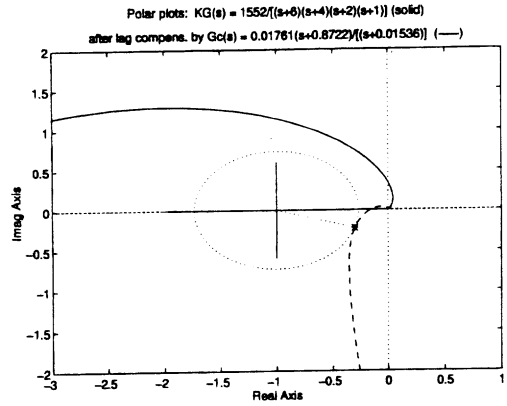

Step response: KG(s) = 72.5(s+2)/[s^2(s^2+4s+29)]
after lead compensation using RD design method.
RD design Gc(s) = 113.7(s+2.862)/[(s+325.3)]
Traditional lead design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

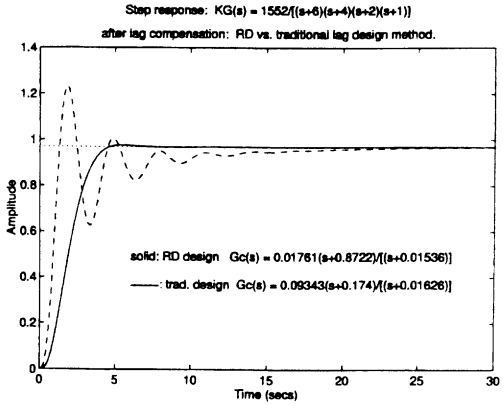

Polar plots: KG(s) = 72.5(s+2)/[s^2(s^2+4s+29)] (solid)
after lag compens. by Gc(s) = 0.7173(s+0.3753)/[(s+0.2692)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 72.5(s+2)/[s^2(s^2+4s+29)]
after lag compensation: RD vs. traditional lag design method.
Amplitude
Time (secs)
solid: RD design Gc(s) = 0.7173(s+0.3753)/[(s+0.2692)]
——: trad. design Gc(s) = 0.6199(s+0.2336)/[(s+0.1448)]

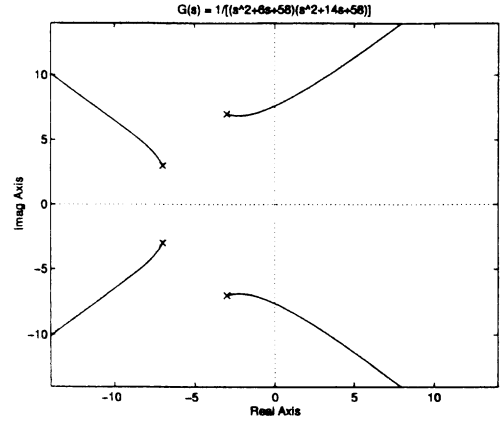
TF 9

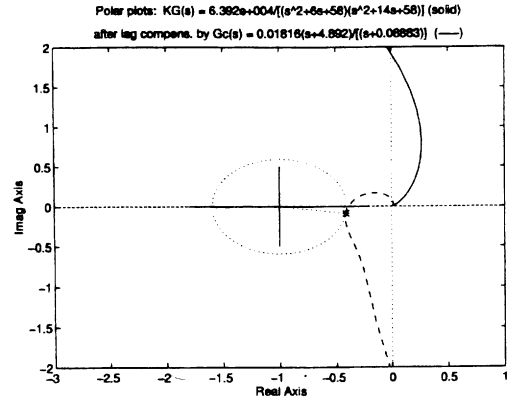

G(s) = 1/[(s+6)(s+4)(s+2)(s+1)]
Imag Axis
Real Axis

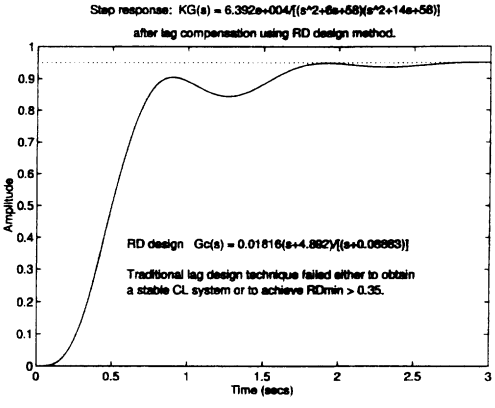

Polar plots: KG(s) = 1552/[(s+6)(s+4)(s+2)(s+1)] (solid)
after lag compens. by Gc(s) = 0.01761(s+0.8722)/[(s+0.01536)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 1552/[(s+6)(s+4)(s+2)(s+1)]
after lag compensation: RD vs. traditional lag design method.
Amplitude
Time (secs)
solid: RD design Gc(s) = 0.01761(s+0.8722)/[(s+0.01536)]
——: trad. design Gc(s) = 0.09343(s+0.174)/[(s+0.01626)]

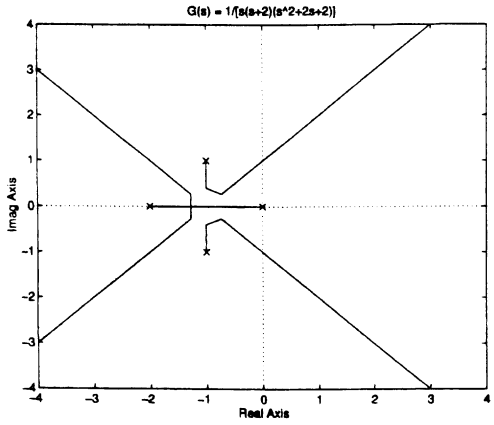
TF 10

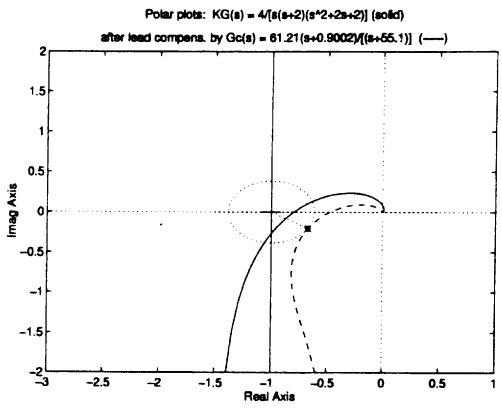

G(s) = 1/[(s^2+6s+58)(s^2+14s+58)]
Imag Axis
Real Axis

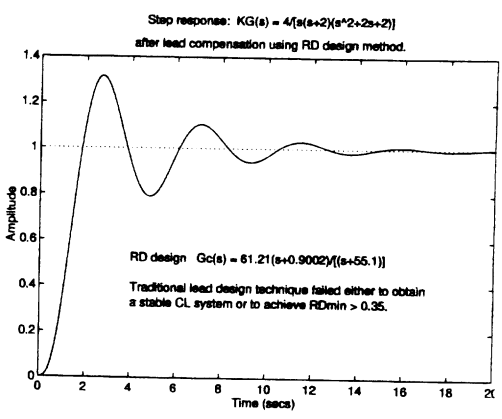

Polar plots: KG(s) = 6.392e+004/[(s^2+6s+58)(s^2+14s+58)] (solid)
after lag compens. by Gc(s) = 0.01816(s+4.892)/[(s+0.06883)] (——)
Imag Axis
Real Axis

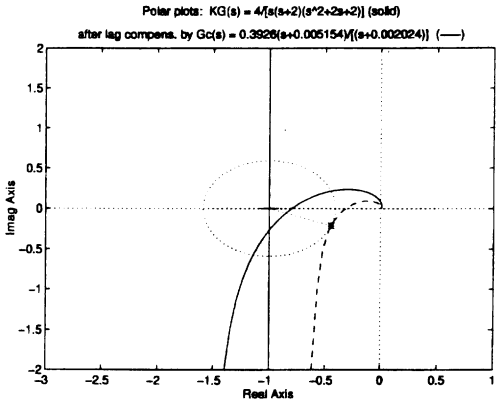

Step response: KG(s) = 6.392e+004/[(s^2+6s+58)(s^2+14s+58)]
after lag compensation using RD design method.
Amplitude
Time (secs)
RD design Gc(s) = 0.01816(s+4.892)/[(s+0.06883)]
Traditional lag design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.

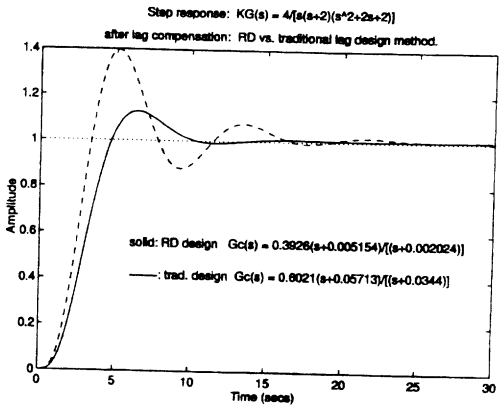
TF 11


G(s) = 1/[s(s+2)(s^2+2s+2)]
Imag Axis
Real Axis

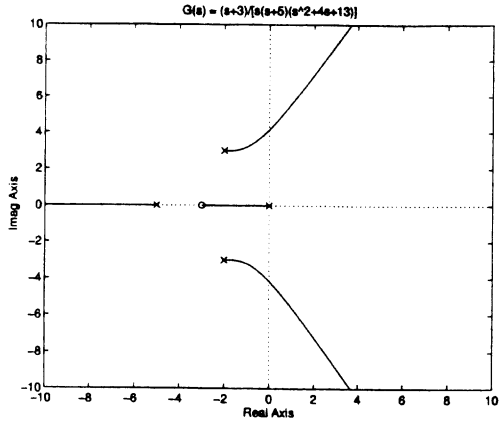

Polar plots: KG(s) = 4/[s(s+2)(s^2+2s+2)] (solid)
after lead compens. by Gc(s) = 61.21(s+0.9002)/[(s+55.1)] (——)
Imag Axis
Real Axis

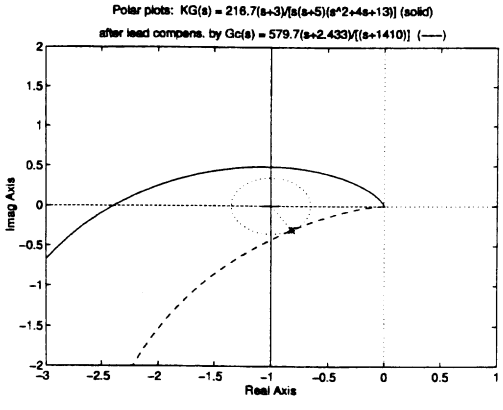

Step response: KG(s) = 4/[s(s+2)(s^2+2s+2)]
after lead compensation using RD design method.
RD design Gc(s) = 61.21(s+0.9002)/[(s+55.1)]
Traditional lead design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

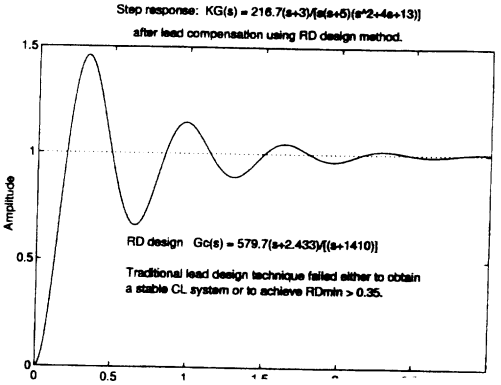

Polar plots: KG(s) = 4/[s(s+2)(s^2+2s+2)] (solid)
after lag compens. by Gc(s) = 0.3926(s+0.005154)/[(s+0.002024)] (——)
Imag Axis
Real Axis

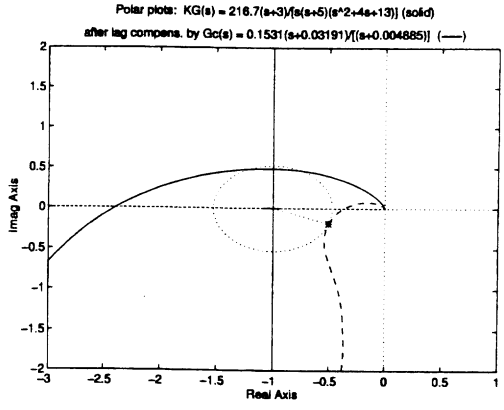

Step response: KG(s) = 4/[s(s+2)(s^2+2s+2)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.3926(s+0.005154)/[(s+0.002024)]
——: trad. design Gc(s) = 0.8021(s+0.05713)/[(s+0.0344)]
Amplitude
Time (secs)

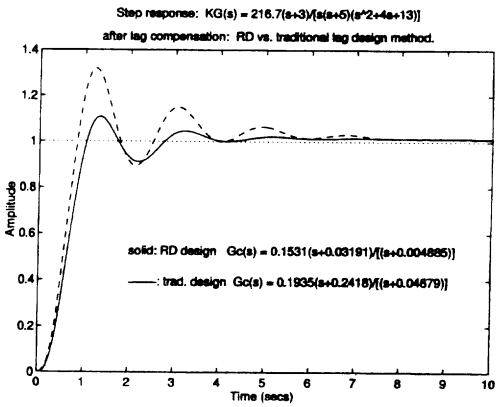
TF 12


G(s) = (s+3)/[s(s+5)(s^2+4s+13)]
Imag Axis
Real Axis

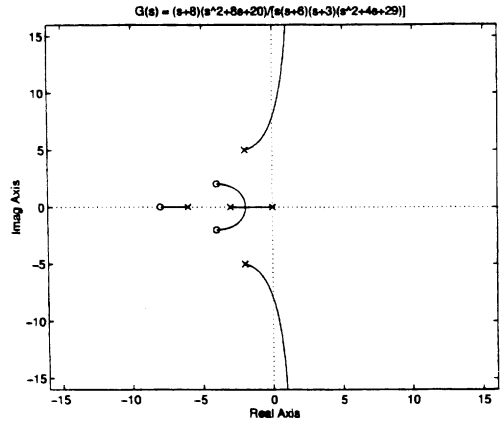

Polar plots: KG(s) = 216.7(s+3)/[s(s+5)(s^2+4s+13)] (solid)
after lead compens. by Gc(s) = 579.7(s+2.433)/[(s+1410)] (——)
Imag Axis
Real Axis

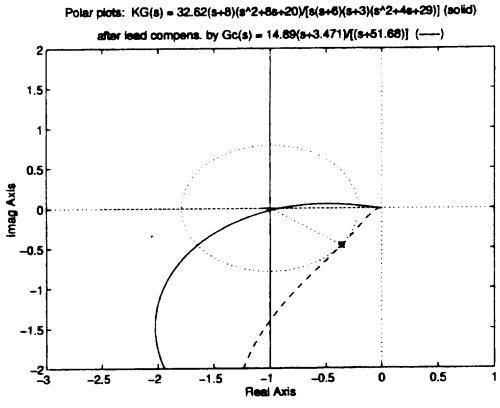

Step response: KG(s) = 216.7(s+3)/[s(s+5)(s^2+4s+13)]
after lead compensation using RD design method.
RD design Gc(s) = 579.7(s+2.433)/[(s+1410)]
Traditional lead design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude

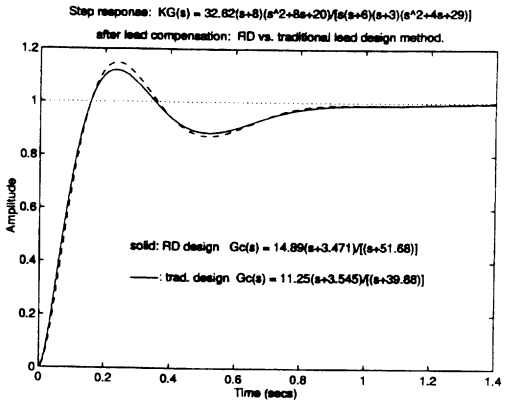

Polar plots: KG(s) = 216.7(s+3)/[s(s+5)(s^2+4s+13)] (solid)
after lag compens. by Gc(s) = 0.1531(s+0.03191)/[(s+0.004885)] (——)
Imag Axis
Real Axis

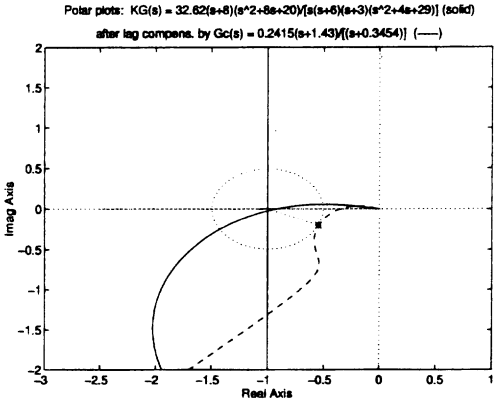

Step response: KG(s) = 216.7(s+3)/[s(s+5)(s^2+4s+13)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.1531(s+0.03191)/[(s+0.004885)]
——: trad. design Gc(s) = 0.1935(s+0.2418)/[(s+0.04679)]
Amplitude
Time (secs)

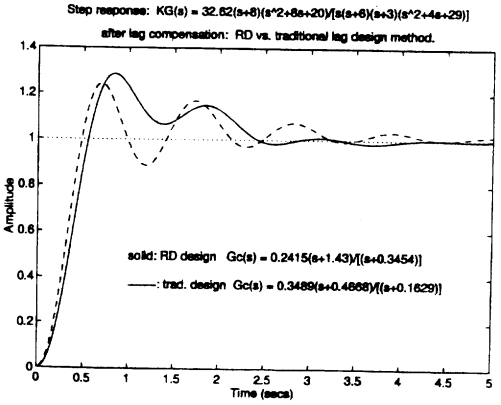
TF 13


G(s) = (s+8)(s^2+8s+20)/[s(s+6)(s+3)(s^2+4s+29)]
Imag Axis
Real Axis

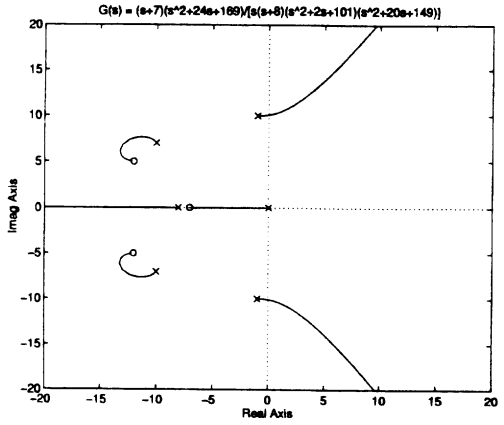

Polar plots: KG(s) = 32.62(s+8)(s^2+8s+20)/[s(s+6)(s+3)(s^2+4s+29)] (solid)
after lead compens. by Gc(s) = 14.89(s+3.471)/[(s+51.68)] (——)
Imag Axis
Real Axis

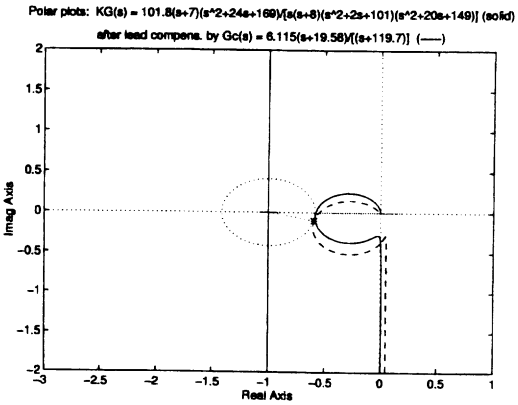

Step response: KG(s) = 32.62(s+8)(s^2+8s+20)/[s(s+6)(s+3)(s^2+4s+29)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 14.89(s+3.471)/[(s+51.68)]
——: trad. design Gc(s) = 11.25(s+3.545)/[(s+39.88)]
Amplitude
Time (secs)

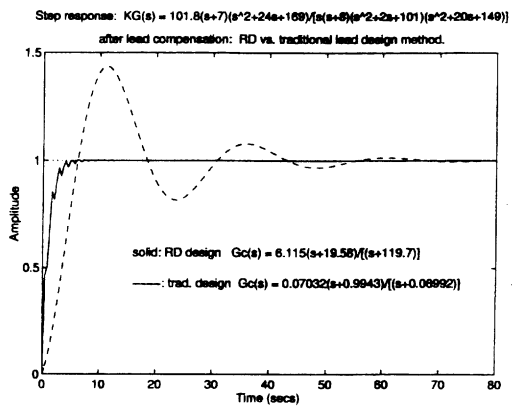

Polar plots: KG(s) = 32.62(s+8)(s^2+8s+20)/[s(s+6)(s+3)(s^2+4s+29)] (solid)
after lag compens. by Gc(s) = 0.2415(s+1.43)/[(s+0.3454)] (——)
Imag Axis
Real Axis

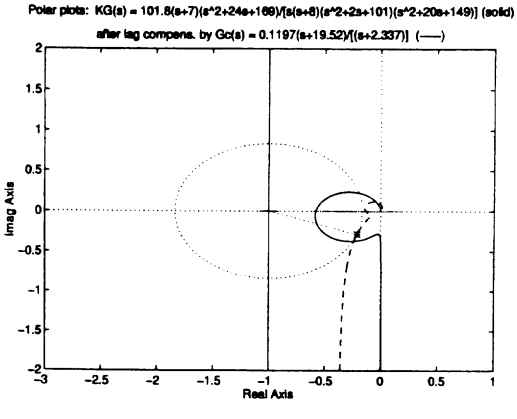

Step response: KG(s) = 32.62(s+8)(s^2+8s+20)/[s(s+6)(s+3)(s^2+4s+29)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.2415(s+1.43)/[(s+0.3454)]
——: trad. design Gc(s) = 0.3489(s+0.4868)/[(s+0.1629)]
Amplitude
Time (secs)

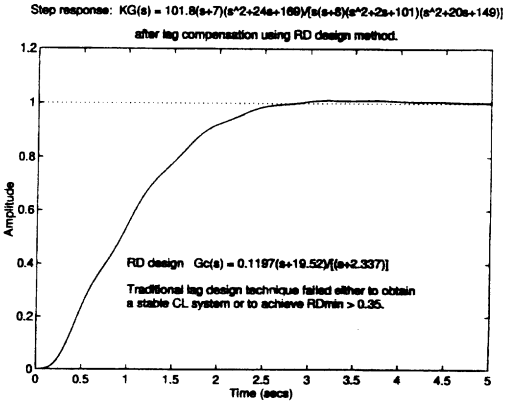
TF 14


G(s) = (s+7)(s^2+24s+169)/[s(s+8)(s^2+2s+101)(s^2+20s+149)]
Imag Axis
Real Axis

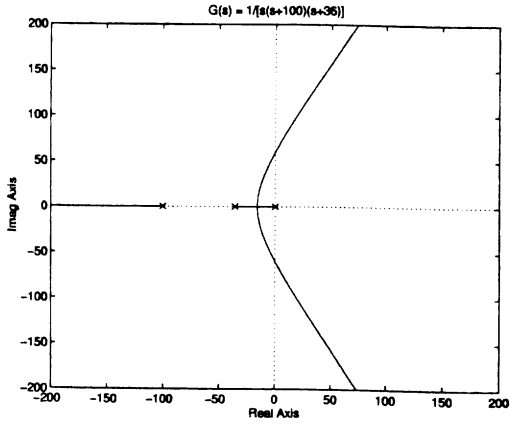

Polar plots: KG(s) = 101.8(s+7)(s^2+24s+169)/[s(s+8)(s^2+2s+101)(s^2+20s+149)] (solid)
after lead compens. by Gc(s) = 6.115(s+19.58)/[(s+119.7)] (——)
Imag Axis
Real Axis

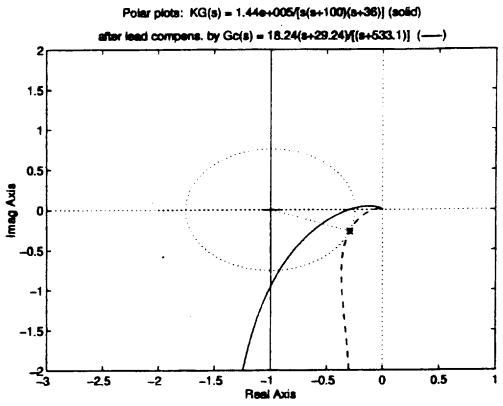

Step response: KG(s) = 101.8(s+7)(s^2+24s+169)/[s(s+8)(s^2+2s+101)(s^2+20s+149)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 6.115(s+19.58)/[(s+119.7)]
——: trad. design Gc(s) = 0.07032(s+0.9943)/[(s+0.06992)]
Amplitude
Time (secs)

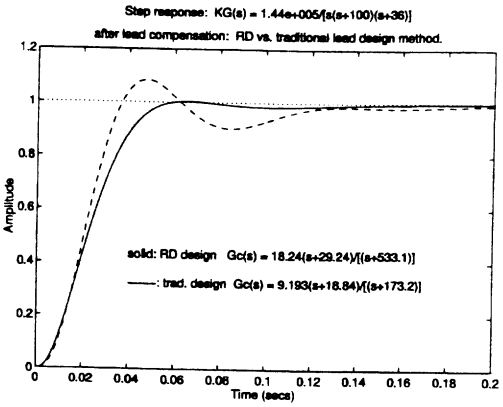

Polar plots: KG(s) = 101.8(s+7)(s^2+24s+169)/[s(s+8)(s^2+2s+101)(s^2+20s+149)] (solid)
after lag compens. by Gc(s) = 0.1197(s+19.52)/[(s+2.337)] (——)
Imag Axis
Real Axis

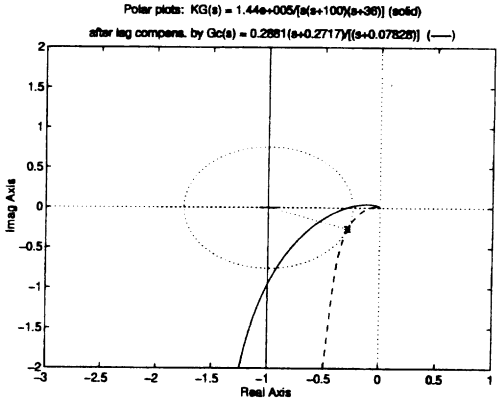

Step response: KG(s) = 101.8(s+7)(s^2+24s+169)/[s(s+8)(s^2+2s+101)(s^2+20s+149)]
after lag compensation using RD design method.
RD design Gc(s) = 0.1197(s+19.52)/[(s+2.337)]
Traditional lag design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

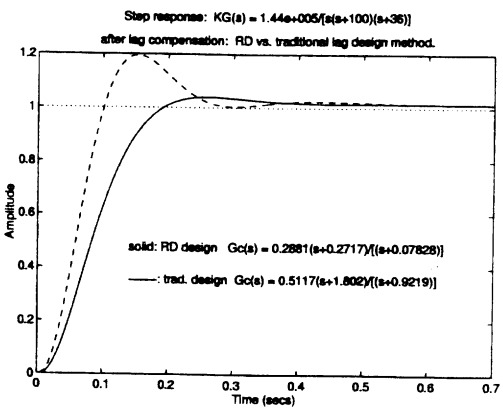
TF 15


G(s) = 1/[s(s+100)(s+36)]
Imag Axis
Real Axis

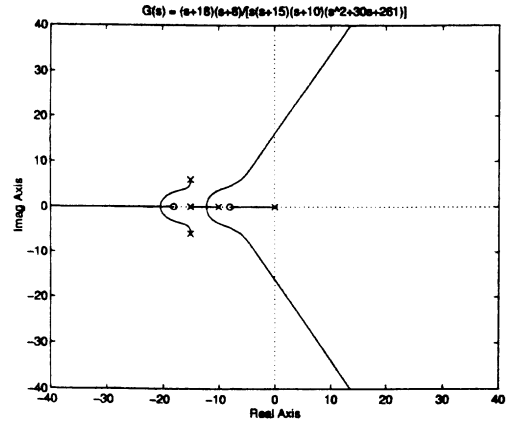

Polar plots: KG(s) = 1.44e+005/[s(s+100)(s+36)] (solid)
after lead compens. by Gc(s) = 18.24(s+29.24)/[(s+533.1)] (——)
Imag Axis
Real Axis

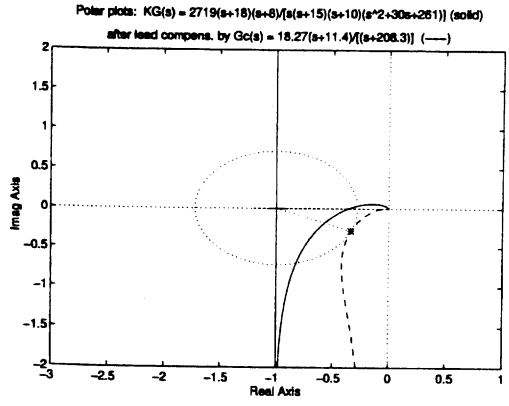

Step response: KG(s) = 1.44e+005/[s(s+100)(s+36)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 18.24(s+29.24)/[(s+533.1)]
——: trad. design Gc(s) = 9.193(s+18.84)/[(s+173.2)]
Amplitude
Time (secs)

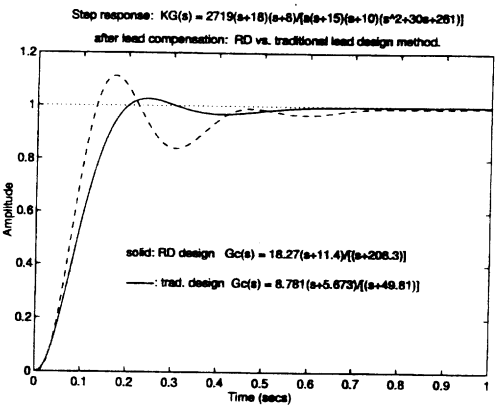

Polar plots: KG(s) = 1.44e+005/[s(s+100)(s+36)] (solid)
after lag compens. by Gc(s) = 0.2881(s+0.2717)/[(s+0.07828)] (——)
Imag Axis
Real Axis

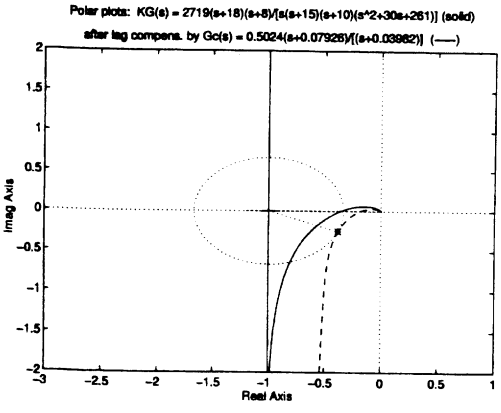

Step response: KG(s) = 1.44e+005/[s(s+100)(s+36)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.2881(s+0.2717)/[(s+0.07828)]
——: trad. design Gc(s) = 0.5117(s+1.802)/[(s+0.9219)]
Amplitude
Time (secs)

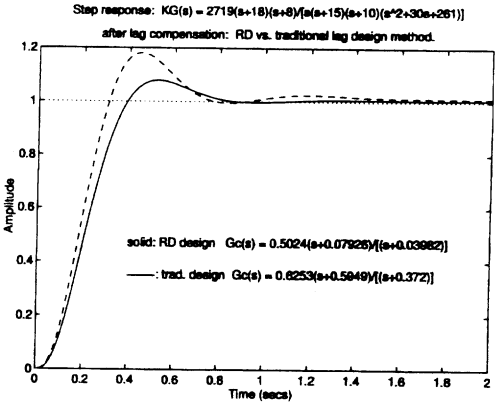
TF 16


G(s) = (s+18)(s+8)/[s(s+15)(s+10)(s^2+30s+261)]
Imag Axis
Real Axis

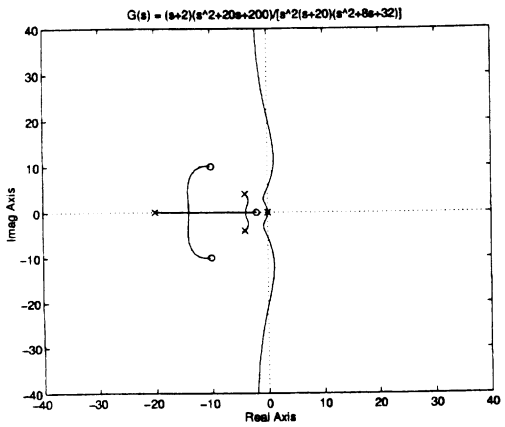

Polar plots: KG(s) = 2719(s+18)(s+8)/[s(s+15)(s+10)(s^2+30s+261)] (solid)
after lead compens. by Gc(s) = 18.27(s+11.4)/[(s+208.3)] (——)
Imag Axis
Real Axis

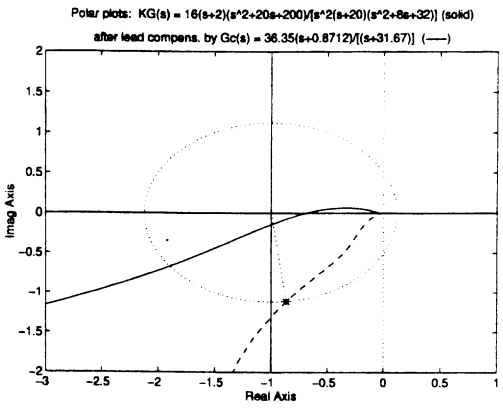

Step response: KG(s) = 2719(s+18)(s+8)/[s(s+15)(s+10)(s^2+30s+261)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 18.27(s+11.4)/[(s+208.3)]
——: trad. design Gc(s) = 8.781(s+5.673)/[(s+49.61)]
Amplitude
Time (secs)

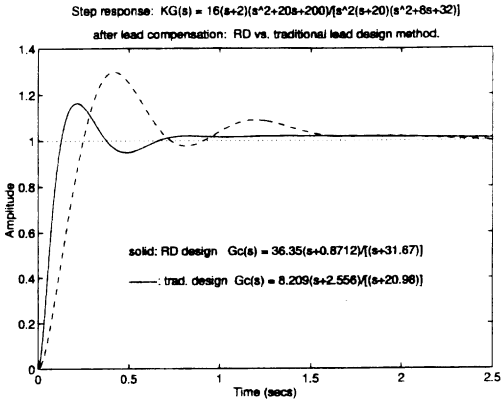

Polar plots: KG(s) = 2719(s+18)(s+8)/[s(s+15)(s+10)(s^2+30s+261)] (solid)
after lag compens. by Gc(s) = 0.5024(s+0.07926)/[(s+0.03962)] (——)
Imag Axis
Real Axis

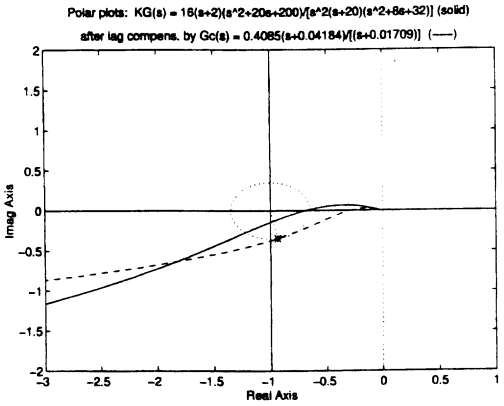

Step response: KG(s) = 2719(s+18)(s+8)/[s(s+15)(s+10)(s^2+30s+261)]
after lag compensation: RD vs. traditional lag design method.
solid: RD design Gc(s) = 0.5024(s+0.07926)/[(s+0.03962)]
——: trad. design Gc(s) = 0.6253(s+0.5849)/[(s+0.372)]
Amplitude
Time (secs)

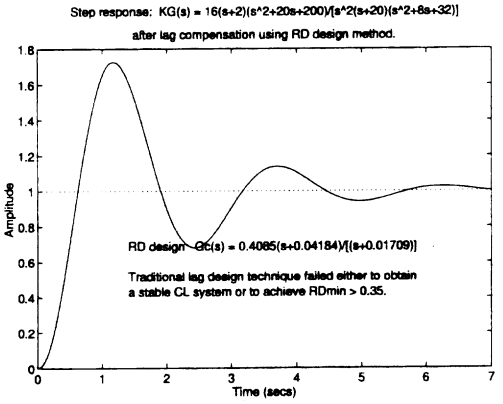
TF 17


G(s) = (s+2)(s^2+20s+200)/[s^2(s+20)(s^2+8s+32)]
Imag Axis
Real Axis


Polar plots: KG(s) = 16(s+2)(s^2+20s+200)/[s^2(s+20)(s^2+8s+32)] (solid)
after lead compens. by Gc(s) = 36.35(s+0.8712)/[(s+31.67)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 16(s+2)(s^2+20s+200)/[s^2(s+20)(s^2+8s+32)]
after lead compensation: RD vs. traditional lead design method.
solid: RD design Gc(s) = 36.35(s+0.8712)/[(s+31.67)]
——: trad. design Gc(s) = 8.209(s+2.556)/[(s+20.98)]
Amplitude
Time (secs)


Polar plots: KG(s) = 16(s+2)(s^2+20s+200)/[s^2(s+20)(s^2+8s+32)] (solid)
after lag compens. by Gc(s) = 0.4085(s+0.04184)/[(s+0.01709)] (——)
Imag Axis
Real Axis


Step response: KG(s) = 16(s+2)(s^2+20s+200)/[s^2(s+20)(s^2+8s+32)]
after lag compensation using RD design method.
RD design Gc(s) = 0.4085(s+0.04184)/[(s+0.01709)]
Traditional lag design technique failed either to obtain
a stable CL system or to achieve RDmin > 0.35.
Amplitude
Time (secs)

4: SEARCH RANGE REFINEMENTS FOR GREATER EFFICIENCY.

An example of increases in search efficiency possible in future versions of the algorithm (such as in a student project) is indicated in reference to Fig. a for lead designs and Fig. b (diagrams on the following pages) for lag designs. Notice that the systems, both uncompensated and compensated, are the same as in Fig. 4 of the paper: $G(s) = (s+5)/[s(s+1.5)(s^2+4s+20)]$ and $K_m = 20.$; some of the details in Fig. 4 have been omitted, and new details have been added for this discussion. These plots show the uncompensated and compensated polar plots for G(s), ω_0, ω_1, ω_0', θ_1, RD_1, RD_{min}, and RD_{min}'. In Fig. a (lead), at a given candidate frequency ω_1, the hatched areas show the possible movement of the polar plot point for ω_1 obtainable by adding a stable, minimum-phase lead compensator. This region is the intersection of the exterior of the $|K_mG(j\omega_1)|$ circle and counter-clockwise (CCW) from the ray at angle $\angle K_mG(j\omega_1)$, because the lead compensator increases both the magnitude and phase of $K_mG(j\omega_1)$. In general, it would be pointless to examine θ_1 (which is negative) more positive than θ_A, where θ_A is the (negative) angle from -1 to the (geometrically determinable) lower intersection of the RD_1 and $|K_mG(j\omega_1)|$ circles; in this case (due to small value selected for K_m for the particular steady-state error requirement) , the circles do not intersect, so the most we could require is that θ_1 be less than θ_B. The general requirement $\theta_1 > \theta_A$ would have to be satisfied because all points on the selected RD_1 circle above θ_A are impossible to achieve with lead compensation, as they all fall within the $|K_mG(j\omega_1)|$ circle. The latter requirement $\theta_1 < \theta_B$ must hold because otherwise θ_1 would involve a phase lag, which a lead compensator cannot provide. Note that choosing $RD_1 > 0.4$ already will keep the compensated polar plot out of the nonrobust circle, as long as RD_{min} is not too much less than RD_1. The reader might arrive at additional constraints on RD_1, ω_1, and θ_1 from this diagram. For example, if many such plots are done for different successful plants/compensated systems, patterns on the approximate relation between optimal θ_1 and ω_1 may emerge. For example, if ω_1 is selected, then the best θ_1 may typically be within a relatively narrow range for most plants and thus the search range for θ_1 may be further tightened. Note: As in the paper, RD_1 has artificially been made significantly larger than RD_{min}' and ω_1 has artificially been moved along the polar plot farther away from ω_0'; the true location of ω_1 is indicated by an X with an arrow pointing into it. Notice that θ_1 in Fig. b now refers to the true location of ω_1, unlike in the paper (as the reader now is sufficiently familiar with the diagram to make that transition). Also note from the discussion above about the limitations on what a lead compensator can do how if K_m is too large, the polar plot will extend beyond -1 and a first-order lead compensator will be unable to stabilize the system.

Similarly, in Fig. b, the region of possible movement by a lag compensator, the hatched region, is within the $|K_mG(j\omega_1)|$ circle and clockwise from the ray at angle $\angle K_mG(j\omega_1)$ because the lag compensator will reduce the gain and phase at $\omega = \omega_1$. It would similarly be pointless to examine designs with θ_1 more negative than θ_C, where θ_C is the (negative) angle from -1 to the intersection of the RD_1 and $|K_mG(j\omega_1)|$ circle closer to the positive real axis because all such θ_1 would be impossible to achieve with a lag compensator. Note: As in the paper, RD_1 has artificially been made significantly larger than RD_{min}' and ω_1 has artificially been moved along the polar plot farther away from ω_0'; the true location of ω_1 is indicated by an X with an arrow pointing into it. Consequently, the artificial location of ω_1 in the compensated system actually violates the rule just established! But notice that the true location of ω_1 is within the hatched area. Notice, however, that θ_1 in Fig. b now refers to the true location of ω_1, unlike in the paper (as the reader now is sufficiently familiar with the diagram to make that transition).

Figure (a)

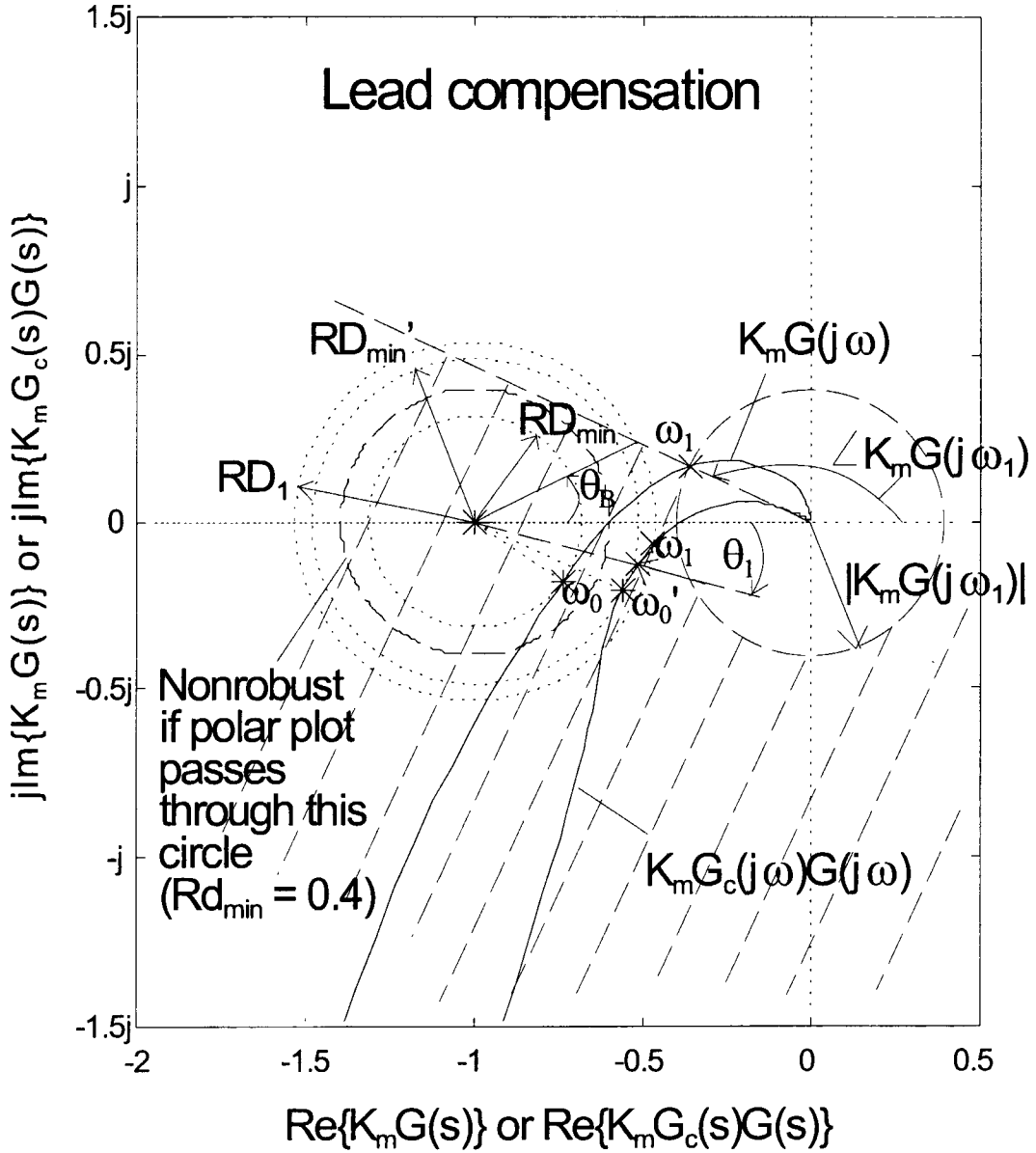
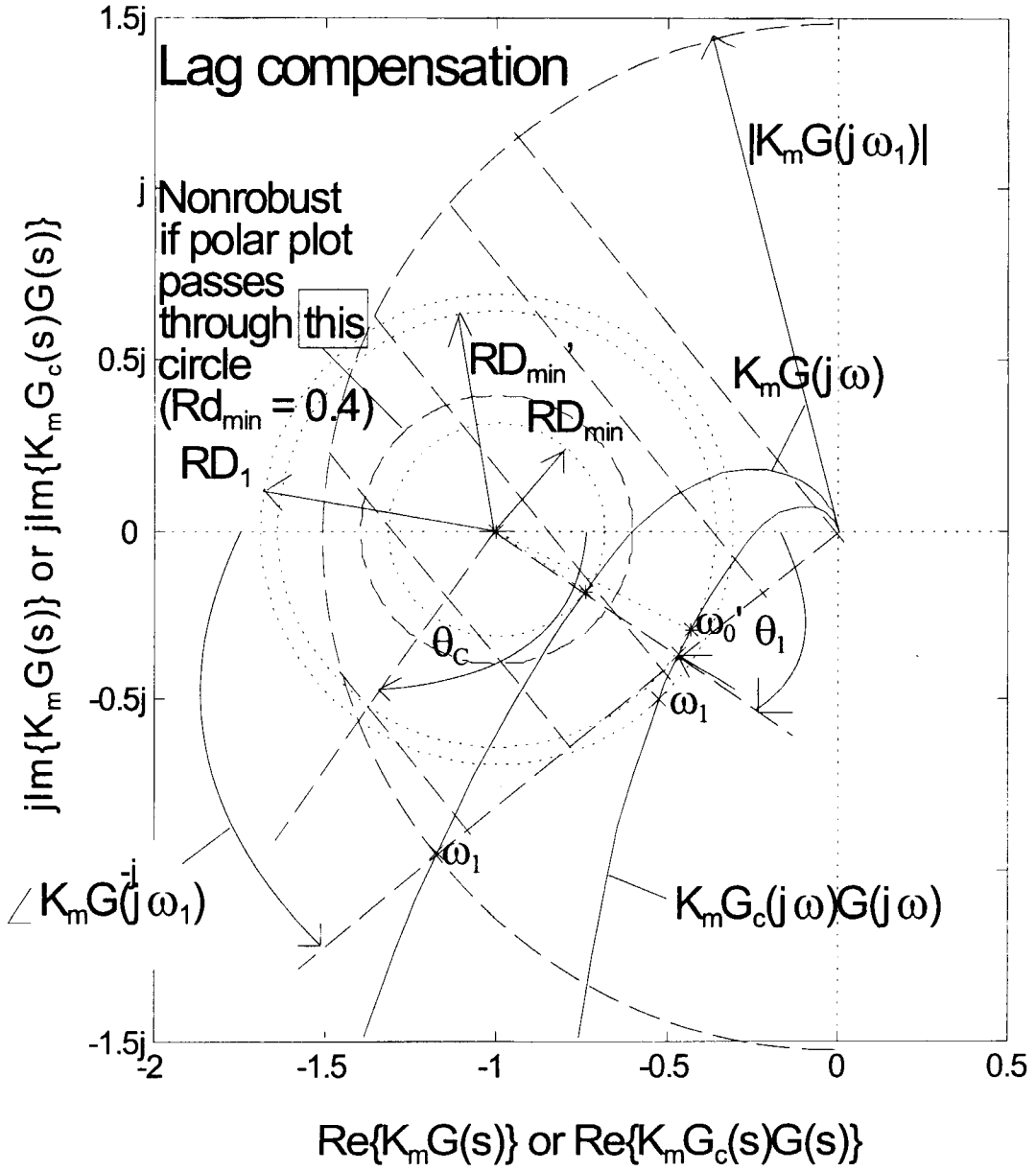

Lead compensation
RD_min'
RD_min
RD_1
θ_B
ω_1
K_mG(jω)
K_mG(jω_1)
|K_mG(jω_1)|
ω_1
θ_1
ω_0
ω_0'
Nonrobust if polar plot passes through this circle (Rd_min = 0.4)
K_mG_c(jω)G(jω)
1.5j
j
0.5j
0
-0.5j
-j
-1.5j
-2
-1.5
-1
-0.5
0
0.5
jIm{K_mG(s)} or jIm{K_mG_c(s)G(s)}
Re{K_mG(s)} or Re{K_mG_c(s)G(s)}

Figure (b)


Lag compensation
Nonrobust if polar plot passes through this circle (Rd_min = 0.4)
|K_m G(j ω_1)|
K_m G(j ω)
RD_min'
RD_min
RD_1
θ_c
ω_0'
θ_1
ω_1
ω_1
∠K_m G(j ω_1)
K_m G_c(j ω) G(j ω)
1.5j
j
0.5j
0
-0.5j
-j
-1.5j
-2
-1.5
-1
-0.5
0
0.5
Im{K_m G(s)} or Im{K_m G_c(s)G(s)}
Re{K_m G(s)} or Re{K_m G_c(s)G(s)}